One way to name the intervals of any EDO from 5 to 72 (with 22edo as an example)

Executive Summary

EdoIntervalNamingMethod.png EdoIntervalNamingMethod.txt

Introduction

When extending the familiar diatonic interval names to equal divisions of the octave having more or less than 12 notes, and having sizes of fifth outside the range of ¼-comma meantone to 12-equal, it is unavoidable that we will either break some of the familiar interval stacking relationships or that we will break the familiar relationship between how dyads are named and how they sound.

One can use interval names based on non-diatonic scales such as the Moment-Of-Symmetry (MOS) scales of the various linear temperaments available within a given EDO. These may have more or less than 7 nominals. In general, such names depart radically from the familiar. The system of this paper avoids revolution in favour of evolution.

One kind of interval naming system has a one-to-one correspondence with the accidentals of a chain-of-fifths based pitch notation such as Sagittal notation, by mapping the following to increasing numbers of the temperament's approximate fifth: d5 m2 m6 m3 m7 P4 P1 P5 M2 M6 M3 M7 A4. I call this a **fifth-based** system [Thanks Jason Yerger]. We need such a system, but it completely breaks the relationship of name to sound.

It is possible to construct a complementary size-based interval naming system based on approximation of just intonation or on fixed ranges of interval size, but this will completely break the familiar interval stacking relationships and their symmetries, when cast into most EDOs.

Instead, the **sound-based** system described in this paper steers a middle course. It preserves the approximate size of the named intervals (to within about +-25 cents) and hence preserves the approximate sound of the named dyads, while also guaranteeing to preserve the symmetries embodied in the following interval stacking relationships (and their octave inversions and extensions). Among other things, they guarantee symmetry within the upper and lower tetrachord or fourth, as well as within the octave.

m3 + M2 = P4
M3 + m2 = P4
M3 + m3 = P5
P4 + m3 = m6
P5 + m2 = m6
P4 + M3 = M6
P5 + M2 = M6
P5 + m3 = m7
P5 + M3 = M7
d5 + A4 = P8
P5 + P4 = P8
m6 + M3 = P8
M6 + m3 = P8

m7 + M2 = P8
M7 + m2 = P8

It does *not* guarantee to preserve the following relationships, except within the ¼-comma meantone to 12-equal range of fifth sizes.

M2 + m2 = m3
M2 + M2 = M3
M3 + M2 = A4
m3 + m3 = d5
P4 + m2 = d5
P4 + M2 = P5
A4 + m2 = P5
d5 + M2 = m6
A4 + m3 = M6
P4 + P4 = m7
d5 + M3 = m7
m6 + M2 = m7
M6 + m2 = m7
A4 + P4 = M7
M6 + M2 = M7

So the sound-based system of this paper is not based purely on interval size, but bends a little in the direction of the fifths-based system and hence the diatonic function of the interval. So in addition to reducing the cognitive load on the user, by means of the many symmetries it retains, this system approximates the context-dependence of the way we hear interval categories.

The names

In accordance with a suggestion by Kite Gedraitis, we reserve the qualifiers: major, minor, diminished and augmented for the fifths-based names, and instead we use: majorish, minorish, dimmish and auggish. And we add: neutral, sub, super, narrow and wide. Narrow and wide are only needed for EDOs with many notes (≳ 31). Neutral is midway between major and minor. The terms sub, super and neutral have been used with the same meaning they have here, since at least the year 1880, thanks to Alexander Ellis and Jan Land.

I use the following abbreviations in this paper.

Abbreviations				Relative size (degrees of 72-edo)	Notes
N	Neutral	P	Perfect	0	use Pi (Perfectish) only if different from fifths-based
n	narrow	W	Wide	1	narrow/wide are not used for EDOs ≲ 31
mi	minorish	Mi	Majorish	2	
s	sub	S	Super	3	
sm	subminor	SM	Supermajor	5	
di	dimmish	Ai	Auggish	5	

I note that “small” and “Large” or “lesser” and “Greater” might have been used instead of “narrow” and “Wide”, except that both have problems with their abbreviations. “s” cannot be used as an abbreviation for “small”, since it is already used for “sub”, and “l” (lowercase “L”) should not be used because it is too easily confused with the digit 1.

The method

To determine the names for the intervals of an EDO having e divisions to the octave, we first calculate the number of degrees in the EDO's best fifth as

$f = \text{Round}(e * 701.955 / 1200)$, where “Round()” means “round to the nearest whole number”.

For example, in 22edo, $e = 22$ and $f = 13$.

The points of reference for the seven interval name classes (unisons through sevenths) are the Perfect and Neutral intervals, arranged in two chains of the EDO’s best fifths, offset by half a fifth as follows.

	P4	---	P1	---	P5	
N2	---	N6	---	N3	---	N7

If f is an odd number, the EDO will not *contain* any neutral intervals, but the neutrals will still be used as points of reference, midway between two notes, for naming the other intervals.

We then draw a circle representing an octave, with marks around its circumference representing the notes of the EDO. You can see such a diagram about halfway through this document.

We mark P1 at the top, count f divisions clockwise and mark P5.
From P1, we count f divisions anticlockwise and mark P4.
From P1, we count $f/2$ divisions clockwise and mark N3.
From P5, we count $f/2$ divisions clockwise and mark N7.
From P1, we count $f/2$ divisions anticlockwise and mark N6.
From P4, we count $f/2$ divisions anticlockwise and mark N2.

For 22edo, $f/2 = 6.5$.

We then use the table below to choose qualifiers corresponding to various numbers of degrees of the EDO, on either side of the perfect and neutral intervals, so as to fill in the gaps, and further to provide alternative names for most intervals.

For most EDOs 31 and lower, it should be possible to choose the names from those which are highlighted and do not involve the terms narrow or Wide. To name the intervals of 72edo we need to use *all* of the qualifiers shown in the diagram. For other EDOs we need to determine, for each qualifier, which degree (or half-degree) of the EDO it is closest to, and could therefore be validly used for. To do this we apply the following formula to each degree of 72edo (d_{72}).

$d_e = \text{Round}(d_{72} * e * 2 / 72) / 2$

The results of this for 22edo are shown in the rightmost column.

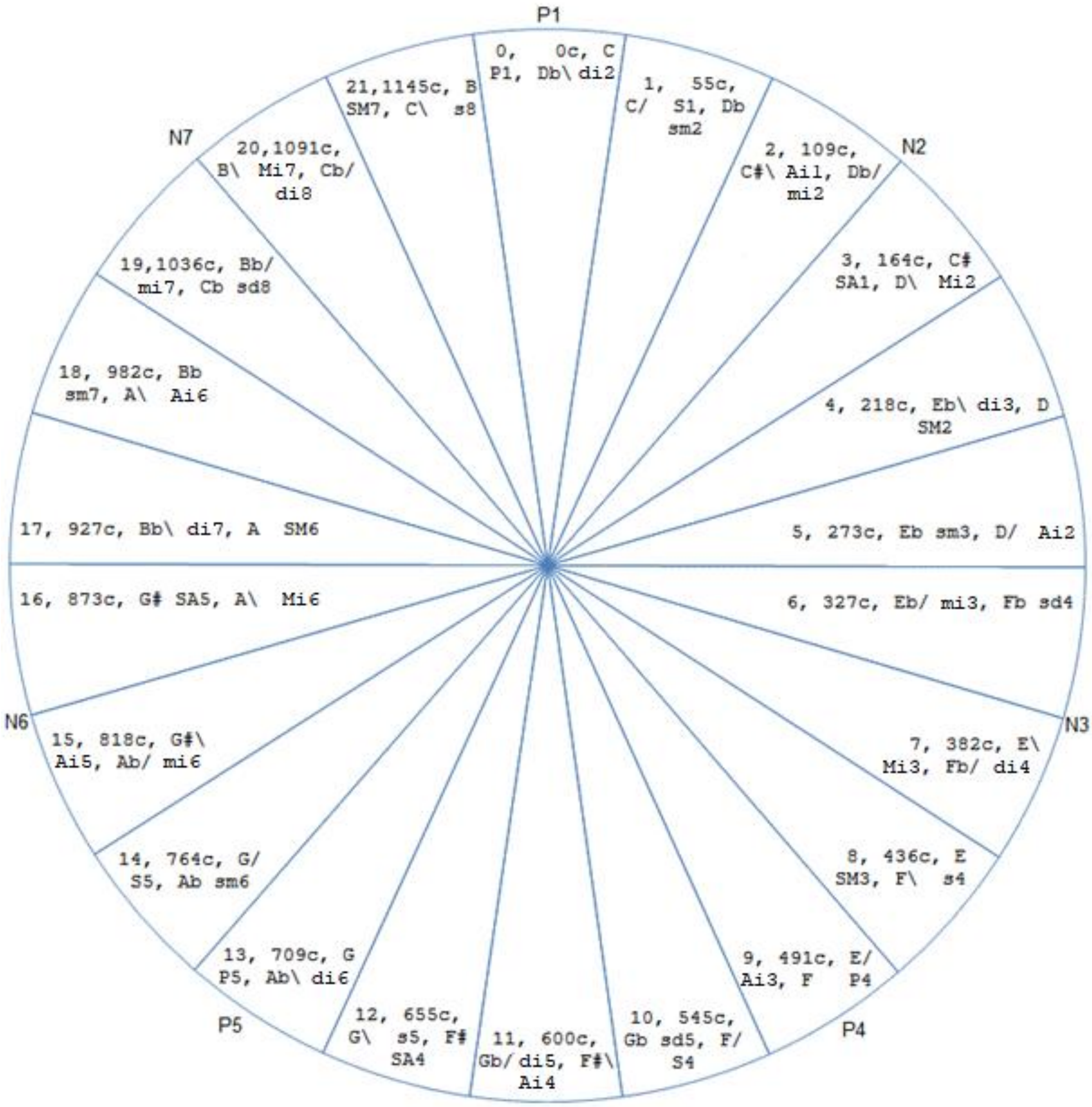
		unisons				seconds				
Degrees		fourths				thirds			Degrees	
of		fifths				sixths			of	
72edo		octaves				sevenths			72edo	22edo
12				WWAA			AA		12	3.5
11				WAA				WSA	11	3.5
10			AA				SA		10	3
9				WSA				nSA	9	3
8			SA				WA		8	2.5
7				nSA		Ai			7	2
6			WA				WSM		6	2
5		Ai				SM			5	1.5
4			WS				nSM		4	1
3		S				WM			3	1
2			nS		Mi				2	0.5
1		W				WN			1	0.5
0	P				N				0	0
-1		n				nN			-1	-0.5
-2			Ws		mi				-2	-0.5
-3		s				nm			-3	-1
-4			ns				Wsm		-4	-1
-5		di				sm			-5	-1.5
-6			nd				nsm		-6	-2
-7				Wsd		di			-7	-2
-8			sd				nd		-8	-2.5
-9				nsd				Wsd	-9	-3
-10			dd				sd		-10	-3
-11				ndd				nsd	-11	-3.5
-12				nndd			dd		-12	-3.5

If the fifth-size *f* is an even number, we use the whole numbers of degrees to look up the qualifiers for both the perfect and the imperfect intervals. If the fifth size is odd we use the whole numbers only for the perfect intervals, and we use the half numbers for the imperfect intervals. When more than one qualifier is valid, we choose the simplest set for which the parts add logically. For example, if S (super) is 1 degree and Ai (Auggish) is 2 then SA (SuperAuggish) should be 3. Be aware that in the case of the imperfect intervals, for the purpose of such arithmetic, Augmented is really Augmented Major (AM) and diminished is really diminished minor (dm), but the “Major” and “minor” parts are dropped by long-standing convention. If such a set contains qualifiers beginning with n for narrow or W for Wide, but dropping these parts would cause no ambiguity, then they should be dropped. This appears to be possible for most EDOs 31 or less.

For the perfect intervals in 22edo we initially obtain the sequence P S WA WSA (0 1 2 3) and we drop the W’s to obtain P S Ai SA (0 1 2 3). The full sequence for the perfect intervals is then sd di s P S Ai SA (-3 -2 -1 0 1 2 3). So we label the intervals either side of P4 as sd4 di4 s4 P4 S4 Ai4 SA4, and similarly for P5 and P1 (which is also P8). See the circular diagram below.

For the imperfect intervals we look at the half degrees and initially obtain the sequence (N) Mi SM WA WSA (0 0.5 1.5 2.5 3.5). I put N in parenthesis here because it is not an interval of the tuning, but merely a reference point. We drop the W’s to obtain (N) Mi SM Ai SA. Note that, for the purpose of the degree arithmetic, this is effectively (N) Mi SM AM SAM. The full sequence for the imperfect intervals is then sd di sm mi (N) Mi SM Ai SA (-3.5 -2.5 -1.5 -0.5 0 0.5 1.5 2.5 3.5). So we label the intervals either side of N3 as sd3 di3 sm3 mi3 (N3) Mi3 SM3 Ai3 SA3, and similarly for N2, N6 and N7.

The diagram below gives the number of 22edo degrees and the number of cents in each interval, and two names for each interval, each preceded by Sagittal notation, in the short-ASCII encoding, for the pitch that will make that interval with a C below it. “\” may be pronounced “pao” or “down” and “/” may be pronounced “pai” or “up”. In long-ASCII encoding these are \! and /|. In a proper Sagittal font they appear as half arrows with a left barb only, down and up.



22edo Interval names

The same information is presented in table form below.

22edo alternative pitch spellings and corresponding sound-based interval names

```
 0,   0c, C     P1, Db\ di2
 1,  55c, C/    S1, Db  sm2
 2, 109c, C#\ Ai1, Db/ mi2
 3, 164c, C#  SA1, D\  Mi2
 4, 218c, Eb\ di3, D   SM2
 5, 273c, Eb  sm3, D/  Ai2
 6, 327c, Eb/ mi3, Fb  sd4
 7, 382c, E\  Mi3, Fb/ di4
 8, 436c, E   SM3, F\   s4
 9, 491c, E/  Ai3, F    P4
10, 545c, Gb  sd5, F/   S4
11, 600c, Gb/ di5, F#\ Ai4
12, 655c, G\   s5, F#  SA4
13, 709c, G    P5, Ab\ di6
14, 764c, G/   S5, Ab  sm6
15, 818c, G#\ Ai5, Ab/ mi6
16, 873c, G#  SA5, A\  Mi6
17, 927c, Bb\ di7, A   SM6
18, 982c, Bb  sm7, A\  Ai6
19,1036c, Bb/ mi7, Cb  sd8
20,1091c, B\  Mi7, Cb/ di8
21,1145c, B   SM7, C\   s8
22,1200c, B/  Ai7, C    P8
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Defending the thesis

It is an important feature of this system that, within any given EDO, there is a one-to-one mapping between accidentals in the pitch notation and qualifiers in the interval naming. But this is not the same as the mapping as for the fifths-based or chain-of-fifths interval names, which have the same mapping for all EDOs. For example:

For all 22edo interval classes that admit of "minorish" and "majorish" qualifiers we have the following mapping from pitch notation to sound-based names:

b\ dimmish
b   subminor
b/ minorish
\ majorish
   supermajor
/ auggish

For all 22edo interval classes that admit of "perfect" qualifiers we have:

b   subdimmish
b/ dimmish
\ sub
    perfect
/ super
#\ auggish
superauggish

This is of course different from their mapping in 12edo, but there is no reason why this mapping must remain constant across all temperaments. Different mappings are what different temperaments are all about. The very fact that the seconds, thirds, sixths and sevenths with no accidental are no longer major but supermajor, and the unisons, fourths and fifths with a flat and sharp are no longer diminished and augmented but subdimmish and superauggish, tells us that we are not in meantone Kansas anymore. It tells us that we are over the rainbow in superpythagorean land.

It may seem that I'm being inconsistent in (a) promoting EDO pitch notations where the 7 nominals are in a *single* chain of fifths (having codeveloped Sagittal notation) and (b) promoting an interval naming system where the 7 middle-of-class intervals are in *two* chains of fifths a half-fifth apart. But in fact the two systems have very different requirements. The fact that a 7-note linear scale has (in general) 13 different intervals makes it clear that pitches and intervals are very different things.

A pitch notation needs to deal with modulation, so it needs to be very uniform indeed. We have the example of Johnston notation whose nominals are based on the JI major scale. It is a complete nightmare under modulation. The nominals of a pitch notation must be linear, and if we wish to maintain some connection with standard pitch notation, the generator must be a fifth.

An interval naming scheme on the other hand is completely unaffected by modulation. When the pitches modulate, the intervals stay exactly the same. And the conventional naming system already has two superclasses of intervals - the perfects and the imperfects (major/minors). By putting them into two different chains of fifths we end up with chains that are so short that the size of the intervals at the extremes is not so greatly affected by the change in the size of the fifths. The thirds and sixths change only half as much as the fifths do, and the seconds and sevenths only one and a half times as much. This allows us to keep close to the conventional sizes for all the interval names.

Some other EDOs

Below, I give interval names obtained by this method for some other EDOs, in an abbreviated form. I have omitted alternative names for most intervals, but when included, they are joined by a slash. For larger EDOs I have only shown intervals to the half-octave. The rest can be determined by symmetry.

5 edo P1 Ai2/di3 P4 P5 Ai6/di7
7 edo P1 N2 N3 P4 P5 N6 N7
11 edo P1 sm2 N2 N3 SM3 P4 P5 sm6 N6 N7 SM7 * (better named as a subset of 22)
13 edo P1 N2 SM2 sm3 N3 P4 Ai4 di5 P5 N6 SM6 sm7 N7 * (better named as a subset of 26)
15 edo P1 mi2 Mi2 Ai2/di3 mi3 Mi3 P4 Ai4 di5 P5 mi6 Mi6 Ai6/di7 mi7 Mi7
16 edo P1 Ai1/di2 mi2 Mi2/di3 mi3 Mi3 Ai3/di4 P4 Ai4/di5 P5 Ai5/di6 mi6 Mi6 Ai6/mi7 Mi7 Ai7/di8
17 edo P1 Ai1/sm2 N2 SM2 sm3 N3 SM3/di4 P4 Ai4/sd5 SA4/di5 P5 Ai5/sm6 N6 SM6 sm7 N7 SM7/s8
19 edo P1 sm2 mi2 Mi2 SM2/sm3 mi3 Mi3 SM3/s4 P4 S4/sd5 SA4/s5 P5 S5/sm6 mi6 Mi6 SM6/sm7 mi7 Mi7 SM7
21 edo P1 S1/sm2 mi2 N2 Mi2/sm3 mi3 N3 Mi3 s4/SM3 P4 S4/di5 s5/Ai4 P5 S5/sm6 mi6 N6 Mi6/sm7 mi7 N7 Mi7 SM7
22 edo P1 sm2 mi2 Mi2 SM2 sm3 mi3 Mi3 SM3 P4 S4 Ai4/di5 s5 P5 sm6 mi6 Mi6 SM6 sm7 mi7 Mi7 SM7
23 edo P1 S1 sm2 mi2 Mi2/di3 sm3 mi3 Mi3 SM3/di4 s4/Ai3 P4 S4 s5 P5 S5 Ai5/sm6 mi6 Mi6 sm7 mi7 Mi7 SM7 s8

26 edo P1 S1 sm2 mi2 Mi2 SM2/di3 sm3 mi3 Mi3 SM3/di4 s4/Ai3 P4 S4 Ai4/di5 s5 P5 S5 sm6 mi6 Mi6 SM6 sm7 mi7 Mi7 SM7 s8
29 edo P1 S1 sm2 nN2 WN2 SM2 WSM2/nsm3 sm3 nN3 WN3 SM3/di4 s4 P4 S4 Ai4 di5
31 edo P1 S1 sm2 mi2 N2 Mi2 SM2 sm3 mi3 N3 Mi3 SM3 s4 P4 S4 Ai4 di5
33 edo P1 nS1 WS1/sm2 mi2 nN2 WN2 Mi2 SM2/sm3 mi3 nN3 WN3 Mi3 SM3/ns4 Ws4 P4 nS4 WS4 ns5
34 edo P1 S1 WS1/sm2 mi2 N2 Mi2 SM2 WSM2/nsm3 sm3 mi3 N3 Mi3 SM3 s4 P4 S4 WS4 Ai4/di5
41 edo P1 nS1 sm2 Wsm2 mi2 N2 Mi2 nSM2 SM2 sm3 Wsm3 mi3 N3 Mi3 nSM3 SM3 Ws4 P4 nS4 S4 Ai4 di5

* The interval names given above for **11edo** and **13edo** are only included to show what the system would generate if applied to such extreme fifth sizes, so you can see why it makes more sense to name their intervals as subsets of 22edo and 26edo respectively.

Why 72?

Why use 72edo as the basis of this nomenclature? Because, outside of the effects of particular melodic scales, the most salient points on the spectrum of dyads are the justly-intoned dyads -- those that correspond to small whole-number ratios of frequency. You can see the proposed names annotated on a harmonic entropy graph here:
http://dkeenan.com/Music/HeWithIntervalNames.png

72edo approximates many of these with uncanny precision and consistency, while bringing all the advantages of a uniformly spaced system. And one that is a multiple of the familiar 12edo.

It is not necessary to know anything about frequency ratios, or just intonation, to use this system. However, for those who are interested, I provide the following table.

```
72edo  Interval
degree name   Ratio
--------------------
 0      P1     1:1
 7     mi2    15:16
 9      N2    11:12
10     WN2    10:11
11     Mi2     9:10
12     WM2     8:9
14     SM2     7:8
16     sm3     6:7
18     nm3    27:32
19     mi3     5:6
21      N3     9:11
22     WN3    13:16
23     Mi3     4:5
25    nSM3    11:14
26     SM3     7:9
30      P4     3:4
33      S4     8:11
35     Ai4     5:7
37     di5     7:10
39      s5    11:16
42      P5     2:3
46     sm6     9:14
45    Wsm6     7:11
49     mi6     5:8
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50	nN6	8:13
51	N6	11:18
53	Mi6	3:5
54	WM6	16:27
56	SM6	7:12
58	sm7	4:7
60	nm7	9:16
61	mi7	5:9
62	nN7	11:20
63	N7	6:11
65	Mi7	8:15
72	P8	1:2

This also helps to explain how it was decided which degrees of 72 would have the shorter names (without narrow or wide). In general the simpler ratio gets the simpler name, but there are some exceptions due to the fact that we want symmetry between intervals and their octave-inversions, as well as wanting seconds to agree with thirds (as inversions within a tetrachord). Any symmetry that can be built into the system reduces the cognitive load on the user.

Further information on this terminology can be found in my earlier paper A note on the naming of musical intervals, and my table of Miracle Interval Names but many ideas in those documents have been superceded above.

-- Dave Keenan, 2015-Aug-27, last updated 2019-Dec-06

Updated 2016-Jun-01, to correct some 72edo degree numbers in the final table and add a note that “small” and “Large” may be substituted for “narrow” and “Wide”.

Updated 2018-Oct-06 to acknowledge Ellis and Land in 1880 instead of Fokker in 1960, deprecate small/Large and lesser/Greater due to problems with their abbreviations, reduce the limit for avoiding narrow/Wide from 34edo to approximately 31edo, correct the punctuation for the 19edo and 21edo lists and add lists for 29, 31, 33, 34 edos. Corrected 17 edo dd5 -> sd5, AA4 -> SA4.

Updated 2019-Dec-06 to make it consistent with what was agreed between Kite Gedraitis and myself in regard to the use of “-ish” suffixes and *X*i symbols for sound-based names where these would otherwise conflict with fifths-based names. See this facebook thread, in particular the sub-thread with over 170 responses.
https://www.facebook.com/groups/497105067092502/permalink/1339878122815188/
But I have not yet made it consistent with some of the other aspects of our agreement there. I think that needs a complete rewrite.

Updated 2019-Dec-14 to include a link to the annotated harmonic entropy graph and correct the nomenclature given for 41-edo, and to include this link to another relevant facebook thread.
https://www.facebook.com/groups/497105067092502/permalink/1691485284321135/
In that thread I agreed we should have a tritone (TT) bucket. I suggested it should be from 575c to 625c.